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Holography, degenerate horizons and entropy

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Abstract

We show that a realization of the correspondence $\mathrm{AdS}_2/\mathrm{CFT}_1$ for near extremal Reissner-Nordström black holes in arbitrary dimensional Einstein-Maxwell gravity exactly reproduces, via Cardy's formula, the deviation of the Bekenstein-Hawking entropy from extremality. We also show that this mechanism is valid for Schwarzschild-de Sitter black holes around the degenerate solution $\mathrm{dS}_2 \times \mathrm{S}^n$. These results reinforce the idea that the Bekenstein-Hawking entropy can be derived from symmetry principles.

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1 Introduction

For a long time it has been a remarkable puzzle to unravel the origin of the Bekenstein-Hawking entropy of a black hole. One would expect that string theory, as a theory of quantum gravity, could offer a microscopic explanation of black hole entropy. However, only recently it has been possible, by applying D-brane techniques, to perform precise calculations which succeed in reproducing the Bekenstein-Hawking entropy for extremal [1, 2] and near-extremal black hole solutions [3] (see also [4, 5]). On the other hand three-dimensional gravity can also be quantized in a consistent way [6, 7] and therefore one can expect to find a statistical interpretation for the BTZ black hole entropy [8]. Carlip showed [9] that, by counting microscopic degrees of freedom of a conformal field theory living in an appropriate boundary, one can exactly reproduce the Bekenstein-Hawking formula for the BTZ black holes. Furthermore, Strominger [10] has also been able to obtain the entropy formula by exploiting the two-dimensional conformal algebra arising as an appropriate symmetry of three-dimensional gravity with a negative cosmological constant [11]. These results suggest that the statistical explanation of the entropy is not too much tied to the details of the quantum theory, but rather to general symmetry properties of the quantum gravity theory. This point of view has been put forward in [12, 13, 14].

The holographic correspondence between gravity on AdS_3 and a two-dimensional conformal field theory, discovered by Brown and Henneaux, was realized in terms of asymptotic symmetries at spatial infinity. This type of realization of the AdS/CFT correspondence [15, 16] was analyzed for the Jackiw-Teitelboim mode of 2D gravity in [17, 18] and further studied in [19] in connection with gravity theories around extremal black hole solutions. The extremal BTZ and four-dimensional Reissner-Nordström black holes possess geometries of the form $\mathrm{AdS}_2\times\mathrm{S}^1$ and $\mathrm{AdS}_2\times\mathrm{S}^2$ respectively. It was shown in [19] that the $\mathrm{AdS}_2/\mathrm{CFT}_1$ correspondence, implemented via asymptotic symmetries, can be used to exactly reproduce the deviation of the Bekenstein-Hawking entropy from extremality. As it was argued in [20], the symmetry algebra of a one-dimensional conformal field theory is just a copy of the Virasoro algebra. The finite-dimensional conformal part of this Virasoro algebra, the SL(2,$\mathbf{R}$) symmetry, is the isometry group of anti-de Sitter space in two dimensions. However, we can alternatively regard the SL(2,$\mathbf{R}$) symmetry as the isometry group of de Sitter space in two space-time dimensions and consider the Virasoro algebra as its natural enlargement to the conformal group in one dimension. One of the aims of this paper is to point out that the realization of the $\mathrm{AdS}_2/\mathrm{CFT}_1$ correspondence in terms of asymptotic symmetries can also be reformulated as a $\mathrm{dS}_2/\mathrm{CFT}_1$ correspondence, providing, in turn, a statistical description of the entropy of Schwarzschild-de Sitter black holes [21] near the degenerate solution (i.e. the Nariai solution [22]), which has the geometry $\mathrm{dS}_2\times\mathrm{S}^2$. This way, the explanation of the entropy for two physically different situations, near extremal Reissner-Nordström and near degenerate Schwarzschild-de Sitter black holes, is similar and seems to indicate the universality of the mechanism. The second goal of this paper is to show that this result is valid in any dimension, thus reinforcing the idea that the Bekenstein-Hawking entropy can be just derived from symmetry considerations.

The paper is structured as follows. In Sect.2 we review, in a parallel way, the Reissner-Nordström and Schwarzschild-de Sitter black hole solutions and the corresponding degenerate limits: the Robinson-Bertotti ($\mathrm{AdS}_2\times\mathrm{S}^2$) [23, 24] and Nariai ($\mathrm{dS}_2\times\mathrm{S}^2$) solutions, respectively. These degenerate solutions represent either black holes of minimum size (for a given electrical charge) or black holes of maximum size (for a given cosmological constant $\Lambda > 0$). In both cases these solutions are stable. The degenerate Reissner-Nordström solution is extremal and the Schwarzschild-de Sitter solution possesses two horizons (the Schwarzschild black hole horizon and the cosmological one) with the same size and the same temperature, thus being in thermal equilibrium. In Sect.3 we shall show, also in a parallel way, that the deviation of the Bekenstein-Hawking entropy of nearly degenerate black holes from the entropy of the degenerate solution can be derived, via Cardy's formula [25], from the Virasoro algebra of asymptotic symmetries. We shall emphasize the fact that this mechanism, already introduced in [19], works for both situations: for asymptotic geometries of the form $\mathrm{AdS}_2\times\mathrm{S}^2$ and also $\mathrm{dS}_2\times\mathrm{S}^2$. In Sect.4 we shall generalize the above results for Reissner-Nordström and Schwarzschild-de Sitter black holes in any dimension. Finally, in Sect.5, we state our conclusions.

2 Degenerate horizons and (A)$\mathrm{dS}_2\times\mathrm{S}^2$ geometries

First of all we shall briefly review the basic facts concerning the emergence of $\mathrm{AdS}_2\times\mathrm{S}^2$ and $\mathrm{dS}_2\times\mathrm{S}^2$ geometries in the near-horizon limit of Reissner-Nordström and Schwarzschild-de Sitter black holes. The Reissner-Nordström (RN) black hole can be described by the metric

$$ds^2 = -V(r)dt^2 + V(r)^{-1}dr^2 + r^2 d\Omega^2\,, \tag{2.1}$$

where

$$V(r) = 1 - \frac{2ml^2}{r} + \frac{q^2 l^4}{r^2}\,, \tag{2.2}$$

q is the electrical charge and $l^2 = G^{(4)}$ is four-dimensional Newton's constant. For $m^2 > q^2$, $V(r)$ has two positive roots corresponding to the inner and external black hole horizons. But in the limit $m^2 \to q^2$ the two roots coincide and the horizons apparently merge. However, this is nothing but an artifact of a poor coordinate choice. In this degenerate case the Schwarzschild coordinates become inappropriate since $V(r) \to 0$ between the two horizons. To see what really happens, let us try

$$\frac{m^2}{q^2} = 1 + \delta^2\,, \tag{2.3}$$

so that the degenerate case is recovered in the limit $\delta \to 0$. We can now define new coordinates ψ and χ by

$$t = \frac{|q|}{\delta}\psi\,, \qquad r = |q|(1 + \delta\sin\chi)\,. \tag{2.4}$$

The resultant metric, a first order in δ, is

$$ds^2 = q^2\left[(\cos^2\chi - \delta\sin\chi\cos 2\chi)d\psi^2 - (1 + \delta\frac{\sin\chi - \sin 3\chi}{2\cos^2\chi})d\chi^2 + d\Omega^2\right]\,, \tag{2.5}$$

and when $\delta = 0$ there is a non-trivial geometry between the horizons

$$ds^2 = -q^2(-\cos^2\chi d\psi^2 + d\chi^2) + q^2 d\Omega^2\,. \tag{2.6}$$

This is the $\mathrm{AdS}_2\times\mathrm{S}^2$ Robinson-Bertotti geometry describing the gravitational field of a covariantly constant electrical field [23, 24]. The transformation (2.4) possesses a remarkable similarity to the Ginsparg-Perry one [26] for the degenerate horizon case in the Schwarzschild-de Sitter (SdS) black hole, where the near-horizon geometry is the $\mathrm{dS}_2\times\mathrm{S}^2$ Nariai geometry [22]

$$ds^2 = \Lambda^{-1}(-\sin^2\chi d\psi^2 + d\chi^2) + \Lambda^{-1}d\Omega^2\,, \tag{2.7}$$

and $\Lambda > 0$ is the cosmological constant. In both (2.6) and (2.7) cases, the geometry is given by the product of two constant-curvature spaces.

We shall now rederive the above results in a more general setting. We start considering the most general spherically symmetrical metric

$$ds^2 = -A^2(r,t)dt^2 + B^2(r,t)dr^2 + D^2(r,t)d\Omega^2 \,. \tag{2.8}$$

If $D(r,t) \neq$ const. in the above metric, we can perform a coordinate transformation $r \to r = D(r,t)$ and, after further coordinate redefinitions, we can write the above metric in the well known form

$$ds^2 = -e^{\tilde{\nu}(r,t)}dt^2 + e^{\tilde{\lambda}(r,t)}dr^2 + r^2 d\Omega^2 \,. \tag{2.9}$$

The only thing that remains to be done is to impose Einstein's equations

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi l^2 T_{\mu\nu} \,, \tag{2.10}$$

where Λ is the cosmological constant. For a cosmological charged body ($A_\mu = (\frac{q}{r}, 0, 0, 0)$) the solution (generalized Birkhoff's theorem) reads as

$$ds^2 = -U(r;\Lambda,q,m)dt^2 + \frac{dr^2}{U(r;\Lambda,q,m)} + r^2 d\Omega^2 \,, \tag{2.11}$$

where

$$U(r;\Lambda,q,m) = 1 - \frac{2ml^2}{r} - \frac{\Lambda}{3}r^2 + \frac{q^2 l^4}{r^2} \,, \tag{2.12}$$

and m is the mass of the black hole. For $\Lambda = q = 0$ we recover the Schwarzschild black hole, for $\Lambda = 0$ the RN black hole and for $q = 0$ the SdS black hole.

It is interesting to comment that, in a different way from the Schwarzschild black hole, the $\Lambda, q \neq 0$ cases possess a richer physics. Whereas for the first case the function $U(r;m)$ only has one zero (the black hole horizon), the presence of new parameters provides more complexity so that the function $U(r;\Lambda,q,m)$ can have different roots, simple or multiple roots, depending in which way we adjust the different parameters. One can find some degenerate cases in which two different horizons become coincident for certain relations between the parameters m, q and Λ. Two simple examples of this feature are the RN and SdS black holes. In the second example there are also two roots (corresponding to the black hole and the cosmological horizon) for $0 < m < \frac{1}{3l^2}\Lambda^{-\frac{1}{2}}$ ($\Lambda > 0$) that become coincident ($r = \Lambda^{-\frac{1}{2}}$) in the limit $m \to \frac{1}{3l^2}\Lambda^{-\frac{1}{2}}$.

Now we consider the case[1] $D^2(r,t) = r_0^2 =$ const. In this case the spacetime decomposes into the product of a two-dimensional manifold and the two-dimensional spherical surface ($M_4 = M_2 \times S^2$); M_2 with coordinates t, r, and S^2 with coordinates θ, φ. We can now proceed in a similar way as the $D(r,t) \neq$ const. case. By means of some coordinate redefinitions we get the following metric

$$ds^2 = -e^{\nu(r,t)}dt^2 + e^{\lambda(r,t)}dr^2 + r_0^2 d\Omega^2 \,. \tag{2.13}$$

Note that both $D(r,t) \neq$ const. and $D(r,t) =$ const. are different solutions not being diffeomorphism connected. Thus, the crucial point is to check the Einstein equations in order to look for possible

[1]This case was already noted in [27].

solutions to the $\nu(r,t)$, $\lambda(r,t)$ functions. As in the $D(r,t) \neq$ const. case we immediately obtain that the above metric should be static. Furthermore it is worthwhile to remark that these kinds of solutions do not always exist. The simplest example emerges in the vacuum $T_{\mu\nu} = 0$ and with a vanishing cosmological constant $\Lambda = 0$. The non-vanishing components of the Einstein tensor are

$$G_0{}^0 = G_1{}^1 = -\frac{1}{r_0^2}\,, \qquad G_2{}^2 = G_3{}^3 = \frac{1}{2}e^{-\lambda}(\nu'' + \frac{1}{2}\nu'^2 - \frac{1}{2}\nu'\lambda')\,, \tag{2.14}$$

and it is immediately noticeable that $G_0{}^0$ and $G_1{}^1$ do not satisfy the vacuum Einstein equations (2.10). Instead, if we consider a non-vanishing stress tensor or a cosmological constant, the situation changes and new solutions for the functions $\nu(r)$ and $\lambda(r)$ appear. We can get more global information about these solutions by taking the trace of (2.10), being the curvature $R = \bar{R} + \hat{R}$, where

$$\bar{R} = -e^{-\lambda}(\nu'' + \frac{1}{2}\nu'^2 - \frac{1}{2}\nu'\lambda')\,, \qquad \hat{R} = \frac{2}{r_0^2}\,, \tag{2.15}$$

are respectively the curvatures of M_2 and S^2. It follows immediately that $R = 4\Lambda$ and then M_2 is also a constant curvature space. Let us analyze two simple and well-known examples.

 # 1. $T_\mu{}^\nu = 0$

In this case the components $G_0{}^0 = G_1{}^1$ satisfy the equations (2.10) for $\Lambda > 0$ being the constant $r_0^2 = \Lambda^{-1}$. Thus M_2 is a positive constant-curvature space and the remaining equations solve for the de Sitter space. The global topology is then $\mathrm{dS}_2\times\mathrm{S}^2$ and the metric is nothing but the Nariai metric (2.7).

 # 2. $\Lambda = 0$

Now $R = 0$, $r_0^2 = q^{-2}$ and equations (2.10) solve for a constant stress tensor. Let us consider the tensor of a constant electrical field

$$T_0{}^0 = T_1{}^1 = -T_2{}^2 = -T_3{}^3 = -\frac{1}{8\pi q^2}\,. \tag{2.16}$$

Thus M_2 becomes the anti-de Sitter space being the global topology $\mathrm{AdS}_2\times\mathrm{S}^2$ and the metric given by (2.6).

In the two above examples the $D(t,r) =$ const. solutions are just the geometries that we found previously around the degenerate horizon configurations in the $D(t,r) \neq$ const. solutions. We shall show this in a more general context in the remaining part of this section. Let us consider again the general static solution (2.9) with $\tilde{\lambda}(r) = -\tilde{\nu}(r) = \ln U(r;m,\xi)$, where m is the mass and ξ's are parameters such as the cosmological constant, electrical charge, etc. The horizons are the roots of $U(r)$. Solutions with horizon degeneracy will be given by $U(r)$ with two or more roots when two neighbouring roots become coincident, in say, r_0, for some determined relations between the parameters $m_0 = m(\xi)$ as it is shown in Fig. 1.

Since r_0 is a double root of $U(r;m,\xi)$ for $m = m_0$, it follows

$$U(r_0;m_0,\xi) = U'(r_0;m_0,\xi) = 0\,, \qquad U''(r_0;m_0,\xi) = -\bar{R}_0\,, \tag{2.17}$$

where primes denote derivatives with respect to the radial coordinate r, and $\bar{R}_0$ is a constant. Now we perform a perturbative transformation around the degenerate radius r_0 by introducing a new pair

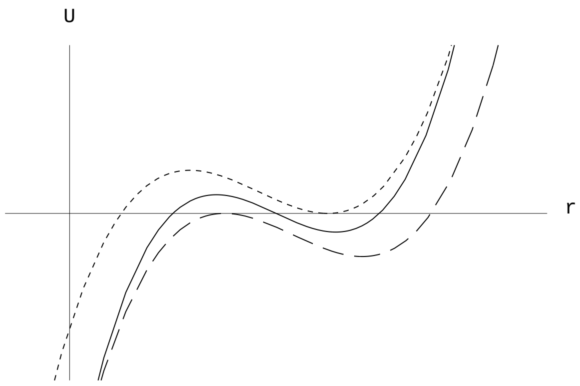

Figure 1: Multi-horizon solutions for different parameter relations. Dotted and dashed lines are degenerate-horizon configurations.

of coordinates $\bar{t}$, $\bar{r}$

$$t = \frac{\bar{t}}{\alpha}\,, \qquad r = r_0 + \alpha\bar{r}\,, \tag{2.18}$$

where $0 < \alpha \ll 1$. We also write $m = m_0(1 + k\alpha^2)$ where k is an arbitrary dimensionless constant being positive for $\bar{R}_0 < 0$ and negative for $\bar{R}_0 > 0$. The degenerate case is recovered when $\alpha = 0$. Expanding in powers of $r - r_0$ in a similar way to what was found in [28], the metric (2.9) turns into

$$ds^2 = -\left(-a^2 - \frac{\bar{R}_0}{2}\bar{r}^2 + \mathcal{O}(\alpha)\right)d\bar{t}^2 + \frac{d\bar{r}^2}{-a^2 - \frac{\bar{R}_0}{2}\bar{r}^2 + \mathcal{O}(\alpha)} + \left(r_0^2 + \mathcal{O}(\alpha)\right)d\Omega^2\,, \tag{2.19}$$

where $a^2 = km_0\partial_m U(r_0, m, \xi)$, and still remains a non-trivial geometry in the near-horizon limit $\alpha \to 0$ with constant curvature $R = \bar{R}_0 + \frac{2}{r_0^2}$. Note that $\bar{R}_0$ is positive (negative) depending on the timelike (spacelike) character of the region between the horizons (see Fig. 1) and, in fact, it can be written as $\bar{R}_0 = \pm\frac{2}{r_0^2}$. Concerning the two examples considered earlier, we have $r_0 = \Lambda^{-\frac{1}{2}}$, $m_0 = \frac{1}{3}\Lambda^{-\frac{1}{2}}$ for the SdS gravity, whereas $r_0 = |q| = m_0$ for the EM gravity.

The existence of a connection between the presence of black hole solutions with horizon degeneracy and (A)dS$_2\times$S^2 decomposed solutions is now clear. The construction of these kinds of solutions from Birkhoff's theorem is associated with the existence of multi-horizon black hole solutions, and they also arise as the near-horizon geometries around degenerate horizons.

3 Holography and entropy of nearly degenerate RN and SdS black holes

In this section we shall explain how the deviation of the Bekenstein-Hawking entropy from extremality for four-dimensional Reissner-Nordström black holes can be derived in terms of the asymptotic symmetries of the corresponding near-horizon geometry. Moreover, we shall also show, in a parallel way, that this mechanism can be used to obtain the deviation of the entropy of SdS black holes from the entropy of the degenerate solution. In both cases the near-horizon geometry, i.e. the leading order metric in power expansion with respect to the parameter α, can be written as

$$ds^2 = -(-a^2 - \frac{\bar{R}_0}{2}\bar{x}^2)d\bar{t}^2 + (-a^2 - \frac{\bar{R}_0}{2}\bar{x}^2)^{-1}d\bar{x}^2 + r_0^2 d\Omega^2\,. \tag{3.1}$$

Assuming the following boundary conditions for the asymptotic expansion of the two-dimensional metric

$$g_{\bar{t}\bar{t}} = \frac{\bar{R}_0}{2}\bar{x}^2 + \gamma_{\bar{t}\bar{t}} + \dots\,, \tag{3.2}$$

$$g_{\bar{t}\bar{x}} = \frac{\gamma_{\bar{t}\bar{x}}}{\bar{x}^3} + \dots\,, \tag{3.3}$$

$$g_{\bar{x}\bar{x}} = -\frac{2}{\bar{R}_0}\frac{1}{\bar{x}^2} + \frac{\gamma_{\bar{x}\bar{x}}}{\bar{x}^4} + \dots\,, \tag{3.4}$$

it is not difficult to see that the infinitesimal diffeomorphisms $\zeta^a(\bar{x},\bar{t})$ preserving the above boundary conditions are

$$\zeta^{\bar{t}} = \epsilon(\bar{t}) - \frac{2}{\bar{R}_0^2\bar{x}^2}\epsilon(\bar{t}) + \mathcal{O}\left(\frac{1}{\bar{x}^4}\right)\,, \tag{3.5}$$

$$\zeta^{\bar{x}} = -\bar{x}\epsilon'(\bar{t}) + \mathcal{O}\left(\frac{1}{\bar{x}}\right)\,, \tag{3.6}$$

where the prime means derivative with respect to the "$\bar{t}$" coordinate, which is a time-like coordinate for AdS_2 ($\bar{R}_0 < 0$) and space-like for dS_2 ($\bar{R}_0 > 0$). The $\mathcal{O}\left(\frac{1}{\bar{x}^4}\right)$ terms in the $\bar{t}$ component are arbitrary and represent the pure gauge transformations. Choosing for instance

$$\zeta^{\bar{t}} = \frac{\alpha^{\bar{t}(\bar{t})}}{\bar{x}^4}\,, \tag{3.7}$$

$$\zeta^{\bar{x}} = \frac{\alpha^{\bar{x}(\bar{t})}}{\bar{x}}\,, \tag{3.8}$$

one can show that $\gamma_{\bar{t}\bar{t}}$, $\gamma_{\bar{x}\bar{x}}$ and $\gamma_{\bar{t}\bar{x}}$ transform as follows

$$\delta\gamma_{\bar{t}\bar{t}} = -\bar{R}_0\alpha^{\bar{x}}\,, \tag{3.9}$$

$$\delta\gamma_{\bar{x}\bar{x}} = -\frac{8}{\bar{R}_0}\alpha^{\bar{x}}\,, \tag{3.10}$$

$$\delta\gamma_{\bar{t}\bar{x}} = \frac{2}{\bar{R}_0}\alpha^{\bar{x}} + 2\bar{R}_0\alpha^{\bar{t}}\,, \tag{3.11}$$

and this implies that one can make the gauge choice

$$\gamma_{\bar{t}\bar{x}} = 0\,. \tag{3.12}$$

Moreover it is just

$$\Theta_{\bar{t}\bar{t}} = \kappa\left(\gamma_{\bar{t}\bar{t}} - \frac{1}{2}\left(\frac{\bar{R}_0}{2}\right)^2\gamma_{\bar{x}\bar{x}}\right)\,, \tag{3.13}$$

where κ is a constant coefficient, the unique gauge invariant quantity and it transforms according to the rule

$$\delta_\epsilon\Theta_{\bar{t}\bar{t}} = \epsilon(\bar{t})\Theta'_{\bar{t}\bar{t}} + 2\Theta_{\bar{t}\bar{t}}\epsilon'(\bar{t}) - \frac{2\kappa}{\bar{R}_0}\epsilon'''(\bar{t})\,. \tag{3.14}$$

Therefore $\Theta_{\bar{t}\bar{t}}$ behaves as the stress-tensor of a (one-dimensional) conformal field theory living on the boundary of (A)dS_2.

We must note that the boundary of AdS_2 is a timelike surface while for dS_2 is spacelike so that the holographic description of the gravitational degrees of freedom of near-extremal Reissner-Nordström and near-degenerate Schwarzschild-de Sitter black holes are physically different. However, mathematically we can treat both situations in a similar way. The Fourier components of the vector fields $\zeta^a\partial_a$, when $\bar{t}$ is considered a compact parameter, close down a Virasoro algebra with a vanishing central charge. Note that for AdS_2 it is natural to consider periodicity in the time coordinate "$\bar{t}$" while for dS_2 the natural periodicity is in the space-like "$\bar{t}$" coordinate. However it is well known that a canonical realization of these types of asymptotic symmetries is allowed to have a non-zero central charge. In fact the expression (3.14)) implies that the Fourier components L^R_n of $\Theta_{\bar{t}\bar{t}}$ (when $0 \leq \bar{t} \leq 2\pi\beta$) are

$$L^R_n = \pm\frac{1}{2\pi\beta}\int_0^{2\pi\beta} d\bar{t}\Theta_{\bar{t}\bar{t}}\beta e^{\pm in\frac{\bar{t}}{\beta}}\,, \tag{3.15}$$

where the positive sign is for $\bar{R}_0 < 0$ (AdS_2) and the negative one for $\bar{R}_0 > 0$ (dS_2), generate a Virasoro algebra

$$i\{L^R_n, L^R_m\} \equiv i\delta_{\epsilon_m}L^R_n = (n-m)L^R_{n+m} + \frac{c}{12}n^3\delta_{n,-m}\,, \tag{3.16}$$

with central charge

$$c = \mp\frac{24}{\bar{R}_0\beta}\kappa\,, \tag{3.17}$$

where the positive constant κ is a coefficient which should be determined by the effective Lagrangian governing the physics near extremality or degeneracy. At this point it is interesting to remark that the integration with respect to the "$\bar{t}$" variable in (3.15), necessary to perform a mode decomposition to get a Virasoro algebra, may seems strange for AdS_2 since then $\bar{t}$ is a time-like coordinate. However for dS_2, $\bar{t}$ is a space-like coordinate and the expression (3.15) fits into the standard procedure of the canonical formalism. This is reminiscent to the fact that, for a two-dimensional conformal field theory, the chirality condition of the stress-tensor components implies that one can transform an integral over a spatial coordinate by an integral over the time variable. Our situation is physically different but it could be an inherent feature of one-dimensional conformal field theory. In fact the degenerate solution of the corresponding Euclidean black holes is, in both cases, of the form $\mathrm{S}^2\times\mathrm{S}^2$, unravelling up to a sign a common origin in the Euclidean sector. Certainly this point merits further analysis[2].

We shall now evaluate the corresponding central charges for both classes of black holes. To this end, and due that the variables $(\bar{t}, \bar{x})$ are the relevant ones, it is quite useful to construct the effective theory describing the effective two-dimensional dynamics. Let us consider the Einstein-Maxwell action with a cosmological constant

$$I^{(4)} = \frac{1}{16\pi G^{(4)}}\int d^4x\sqrt{-g^{(4)}}(R^{(4)} - 2\Lambda + (F^{(4)})^2)\,. \tag{3.18}$$

Imposing spherical symmetry on the metric

$$d\tilde{s}^2_{(4)} = g_{\mu\nu}dx^\mu dx^\nu + l^2\psi^2 d\Omega^2\,, \tag{3.19}$$

where $l^2 = G^{(4)}$, x^μ are the 2D coordinates (t,x) and $d\Omega^2$ is the metric on the two-sphere, and assuming a radial electric field

$$l^{-2}A_\mu = (\frac{q}{r}, 0, 0, 0)\,, \tag{3.20}$$

[2] We thank S. Carlip for stressing us this question.

the above action reduces to

$$I^{(4)} = \frac{1}{4l^2}\int d^2x\sqrt{-g}l^2\psi^2(R + 2|\nabla\psi|^2\psi^{-2} + \frac{2}{l^2\psi^2} - 2q^2\psi^{-4} - 2\Lambda)\,, \tag{3.21}$$

and redefining

$$ds^2 = \sqrt{\phi}d\tilde{s}^2\,, \tag{3.22}$$

$$\phi = \frac{\psi^2}{4}\,, \tag{3.23}$$

we arrive at

$$I^{(4)} = \int d^2x\sqrt{-g}(R\phi + l^{-2}V(\phi))\,, \tag{3.24}$$

where

$$V(\phi) = (4\phi)^{-\frac{1}{2}} - l^2q^2(4\phi)^{-\frac{3}{2}} - l^2\Lambda(4\phi)^{\frac{1}{2}}\,. \tag{3.25}$$

The solutions in terms of the two-dimensional metric $g_{\mu\nu}$ take the form

$$ds^2 = -(J(\phi) - lm)dt^2 + (J(\phi) - lm)^{-1}dr^2\,, \tag{3.26}$$

$$\phi = \frac{r}{l}\,, \tag{3.27}$$

where $J(\phi) = \int_0^\phi d\tilde{\phi}V(\tilde{\phi})$ and in our case

$$J(\phi) = \frac{1}{2}(4\phi)^{\frac{1}{2}} + \frac{1}{2}l^2q^2(4\phi)^{-\frac{1}{2}} - \frac{1}{6}l^2\Lambda(4\phi)^{\frac{3}{2}}\,. \tag{3.28}$$

The degenerate horizons appear for the zeros ϕ_0 of the potential ($V(\phi_0) = 0$). If we perturb around the degenerate radius of coincident horizons

$$m = m_0(1 + k\alpha^2)\,, \tag{3.29}$$

$$t = \frac{\tilde{t}}{\alpha}\,, \tag{3.30}$$

$$r = r_0 + \alpha\tilde{x}\,, \tag{3.31}$$

$$\phi = \phi_0 + \alpha\tilde{\phi}\,, \tag{3.32}$$

we have [28]

$$ds^2 = -(-\frac{\tilde{R}_0}{2}\tilde{x}^2 - km_0l)d\tilde{t} + \frac{d\tilde{x}^2}{-\frac{\tilde{R}_0}{2}\tilde{x}^2 - km_0l} + r_0^2d\Omega^2 + \mathcal{O}(\alpha)\,, \tag{3.33}$$

where

$$\tilde{R}_0 = -\frac{J''(\phi_0)}{l^2}\,. \tag{3.34}$$

We must stress now that the asymptotic symmetries of the effective metric (3.33) are the same as those of the four-dimensional one since the $r - t$ part of both metrics only differs by a constant factor $\sqrt{\phi_0} = \frac{\psi_0}{2}$. In terms of the two-dimensional effective Lagrangian the above expansion reads

$$I^{(4)} = \alpha\int d^2x\sqrt{-g}(R\tilde{\phi} + l^{-2}V'(\phi_0)\tilde{\phi}) + \mathcal{O}(\alpha^2)\,. \tag{3.35}$$

So the leading order is governed by the Jackiw-Teitelboim model [29] (see also [30]). The central charge can be worked out using canonical methods. The full Hamiltonian $\mathcal{H}$ of the theory, to leading order in α, is given by

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{K}\,, \tag{3.36}$$

where $\mathcal{H}_0$ is the bulk Hamiltonian of the Jackiw-Teitelboim theory and $\mathcal{K}$ is the boundary term necessary to have well-defined variational derivatives. Remarkably, the boundary term, after some algebra, turns out to be proportional to the stress-tensor $\Theta_{\tilde{t}\tilde{t}}$

$$K(\epsilon(\tilde{t})) = \epsilon(\tilde{t})\frac{2\alpha}{l}\Big(\gamma_{\tilde{t}\tilde{t}} - \frac{1}{2}\Big(\frac{\tilde{R}_0}{2}\Big)^2\gamma_{\tilde{x}\tilde{x}}\Big)\,, \tag{3.37}$$

where the two-dimensional scalar curvature $\tilde{R}_0$ is related to $\bar{R}_0$ by the expression $\tilde{R}_0 = \bar{R}_0(\phi_0)^{-\frac{1}{2}}$ and making use of the identification [31]

$$K(\epsilon(\tilde{t})) = \epsilon(\tilde{t})\Theta_{\tilde{t}\tilde{t}}\,, \tag{3.38}$$

we can determine the coefficient κ and hence the central charge, which then becomes

$$c = \mp\frac{48\alpha}{l\tilde{R}_0\beta}\,. \tag{3.39}$$

Moreover the value of L_0^R near extremality or degeneracy can also be calculated without difficulty

$$L_0^R = \pm m_0 k\alpha\beta = |q|k\alpha\beta\,, \tag{3.40}$$

and Neveu-Schwartz's generator L_0^{NS} is

$$L_0^{NS} = L_0^R + \frac{c}{24}\,. \tag{3.41}$$

If $L_0^R \gg c$ the asymptotic density of states given by Cardy's formula is

$$\Delta S = 2\pi\sqrt{\frac{cL_0^{NS}}{6}} = 2\pi\sqrt{\frac{8m_0k\alpha^2}{-\tilde{R}_0 l}}\,. \tag{3.42}$$

Let us now check first that this expression exactly accounts for the deviation of the near-extremal Bekenstein-Hawking entropy from extremality. For the Reissner-Nordström black hole we have

$$\tilde{R}_0 = R_0(\frac{2l}{r_0}) = -\frac{4}{l^5|q|^3}\,, \tag{3.43}$$

and

$$\begin{aligned} c &= \frac{12|q|^3l^4\alpha}{\beta}\,, && (3.44)\\ L_0^R &= |q|k\alpha\beta\,, && (3.45)\\ m_0k\alpha^2 &= m - m_0 = \Delta m\,. && (3.46) \end{aligned}$$

So, therefore

$$\Delta S = 2\pi\sqrt{2|q|^3l^4\Delta m}\,, \tag{3.47}$$

and, as it was pointed out in [19], this is just the leading term in the Bekenstein-Hawking entropy

$$S^{BH} = \pi l^2 (|q| + \Delta m + \sqrt{2|q|\Delta m + (\Delta m)^2})^2 \,, \tag{3.48}$$

from the extremal case

$$S_e^{BH} = \pi l^2 |q| \,. \tag{3.49}$$

We shall now analize with more detail the Schwarzschild-de Sitter black hole near degeneracy. The potential function is given by

$$V(\phi) = \frac{1}{2\sqrt{\phi}} - 2l^2 \Lambda \sqrt{\phi} \,, \tag{3.50}$$

so ϕ_0 is

$$\phi_0 = \frac{1}{4l^2\Lambda} \,, \tag{3.51}$$

which corresponds to

$$r_0 = \frac{1}{\sqrt{\Lambda}} \,. \tag{3.52}$$

The curvature $\tilde{R}_0$ is given by

$$\tilde{R}_0 = -\frac{V'(\phi_0)}{l^2} = 4l\Lambda^{\frac{3}{2}} \,, \tag{3.53}$$

which implies that

$$c = \frac{12\alpha}{l^2 \Lambda^{\frac{3}{2}} \beta} \,. \tag{3.54}$$

The Cardy formula leads to ($\Delta m = m - m_0 < 0$)

$$\Delta S = 2\pi \sqrt{-\frac{2\Delta m}{\Lambda^{\frac{3}{2}} l^2}} \,, \tag{3.55}$$

and this is exactly the deviation of the Bekenstein-Hawking entropy from the degenerate solution. Let us see this explicitly. The entropy associated with the cosmological and black hole horizons, located at r_+ and r_- respectively, is given by

$$S_\pm^{BH} = \frac{\pi r_\pm^2}{l^2} \,, \tag{3.56}$$

where r_+, r_- are the two positive roots of the polynomial

$$\frac{\Lambda}{3} r^3 - r - 2l^2 m = 0 \,. \tag{3.57}$$

The solutions are

$$r_+ = \frac{2}{\sqrt{\Lambda}} \cos\frac{\theta}{3} \,, \tag{3.58}$$

$$r_- = \frac{2}{\sqrt{\Lambda}} \cos(\frac{\theta}{3} + \frac{4\pi}{3}) \,, \tag{3.59}$$

where $\cos\theta = -3m\sqrt{\Lambda} l^2$. The degenerate case corresponds to

$$m_0 = \frac{1}{3\sqrt{\Lambda} l^2} \,, \tag{3.60}$$

$$r_0 = \frac{1}{\sqrt{\Lambda}} \,, \tag{3.61}$$

so, if $m \lesssim m_0$

$$\cos\theta \approx -1 - 3\sqrt{\Lambda} l^2 \Delta m\,, \tag{3.62}$$

then

$$r_\pm \approx \frac{1}{\sqrt{\Lambda}} \left(1 \pm \sqrt{-2l^2\sqrt{\Lambda}\Delta m}\right)\,, \tag{3.63}$$

therefore the deviation from the entropy of the degenerate solution is

$$|\Delta S^{BH}_\pm| = \frac{\pi}{l^2} r_0^2 2\sqrt{\frac{-2l^2\Delta m}{\sqrt{\Lambda}}} = \frac{2\pi}{l}\sqrt{\frac{-2\Delta m}{\sqrt{\Lambda^{\frac{3}{2}}}}}\,, \tag{3.64}$$

which agrees with the statistical entropy (3.55).

4 Entropy of near-extremal RN and near-degenerate SdS black holes in any dimension

The aim of this section is to generalize the results of section 3 for arbitrary space-time dimensions. Let us start with the Einstein-Maxwell action with a positive cosmological constant in $(n+2)$ dimensions

$$I^{(n+2)} = \frac{1}{16\pi l^n} \int d^{n+2}x \sqrt{-g^{(n+2)}} \left(R^{(n+2)} - 2\Lambda + (F^{(n+2)})^2\right)\,, \tag{4.1}$$

where l^n is Newton's constant $G^{(n+2)}$. The line element of spherically symmetric solutions is[3]

$$ds^2_{(n+2)} = -U(r)dt^2 + \frac{dr^2}{U(r)} + r^2 d\Omega^2_{(n)}\,, \tag{4.2}$$

where

$$U(r) = 1 - \frac{2l^n m}{r^{n-1}\Gamma_{(n)}} + \frac{l^{2n}q^2}{r^{2(n-1)}\Delta_{(n)}} - \frac{2\Lambda r^2}{n(n+1)}\,, \tag{4.3}$$

$$\Gamma_{(n)} = \frac{n\mathcal{V}^{(n)}}{8\pi}\,, \qquad \Delta_{(n)} = \frac{n}{2(n-1)}\,, \tag{4.4}$$

$\mathcal{V}^{(n)}$ is the area of the unit S^n sphere

$$\mathcal{V}^{(n)} = \frac{n\pi^{\frac{n+1}{n}}}{\Gamma(\frac{n+1}{n})}\,, \tag{4.5}$$

and the electromagnetic field is given by

$$A_\rho = \left(\frac{lq}{(\frac{r}{l})^{n-1}}, 0, \ldots, 0\right)\,, \qquad \rho = 0, 1, \ldots, n+1\,. \tag{4.6}$$

The effective theory of the spherically symmetric sector of (4.1) can be obtained by dimensional reduction. Decomposing the metric as follows

$$ds^2_{(n+2)} = d\hat{s}^2_{(2)}(t,r) + l^2\psi^2(t,r)d\Omega^2_{(n)}\,, \tag{4.7}$$

[3]See [32] for RN solutions.

where $d\Omega^2_{(n)}$ is the metric on the n-sphere, the action (4.1) reduces to[4]

$$\begin{aligned} I^{(n+2)} &= \frac{\mathcal{V}^{(n)}}{16\pi l^n}\int d^2x\sqrt{-\hat{g}}l^n\psi^n\Big(\hat{R}+n(n-1)|\nabla\psi|^2\psi^{-2}+ \\ &\quad \frac{n(n-1)}{l^2}\psi^{-2}-2(n-1)^2q^2\psi^{-2n}-2\Lambda\Big)\,, \end{aligned} \tag{4.8}$$

and performing a redefinition of ψ and a conformal rescaling of the metric

$$\frac{n}{8(n-1)}\psi^n \equiv D(\psi)=\phi\,, \tag{4.9}$$

$$ds^2_{(2)} = \Omega^2(\phi)d\hat{s}^2_{(2)}\,, \tag{4.10}$$

where[5]

$$\Omega^2(\phi)=\frac{n^2}{8(n-1)}\left(\frac{8(n-1)}{n}\phi\right)^{\frac{n-1}{n}}\,, \tag{4.11}$$

we can eliminate the kinetic term in the action (4.8) and then

$$I^{(n+2)}=\frac{1}{2G}\int d^2x\sqrt{-g}(R\phi+l^{-2}V(\phi))\,, \tag{4.12}$$

where

$$G=\frac{n\pi}{(n-1)\mathcal{V}^{(n)}}\,, \tag{4.13}$$

and the potential $V(\phi)$ is given by

$$\begin{aligned} V(\phi) &= (n-1)\left(\frac{8(n-1)}{n}\phi\right)^{\frac{-1}{n}}- \\ &\quad (n-1)\frac{l^2q^2}{\Delta_{(n)}}\left(\frac{8(n-1)}{n}\phi\right)^{\frac{1-2n}{n}}-\frac{2l^2\Lambda}{n}\left(\frac{8(n-1)}{n}\phi\right)^{\frac{1}{n}}\,. \end{aligned} \tag{4.14}$$

The solutions (4.2) transforms into the following solutions of the effective theory (4.12)

$$ds^2_{(2)} = -(J(\phi)-2Glm)dt^2-(J(\phi)-2Glm)^{-1}dx^2\,, \tag{4.15}$$

$$\phi = \frac{x}{l}\,, \tag{4.16}$$

where $J(\phi)=\int_0^\phi d\tilde{\phi}V(\tilde{\phi})$ reads

$$\begin{aligned} J(\phi) &= \frac{n^2}{8(n-1)}\left(\frac{8(n-1)}{n}\phi\right)^{\frac{n-1}{n}}+ \\ &\quad \frac{nl^2q^2}{4}\left(\frac{8(n-1)}{n}\phi\right)^{\frac{1-n}{n}}-\frac{2nl^2\Lambda}{n^2-1}\left(\frac{8(n-1)}{n}\phi\right)^{\frac{n+1}{n}}\,. \end{aligned} \tag{4.17}$$

The degenerate solutions appear for the zeros of the potential

$$V(\phi_0)=J'(\phi_0)=0\,, \tag{4.18}$$

[4]The $q=\Lambda=0$ case was already considered in [33].
[5]See Appendix A for more details.

and the two-dimensional geometry around the degenerate horizon has a constant curvature

$$\tilde{R}_0 = -\frac{J''(\phi_0)}{l^2} \,. \tag{4.19}$$

A canonical analysis leads to the central charge

$$c = \mp\frac{24\alpha}{lG\tilde{R}_0\beta} \,, \tag{4.20}$$

and a value of L_0^R given by

$$L_0^R = \pm m_0 k\alpha\beta \,, \tag{4.21}$$

where we have assumed a periodicity of $2\pi\beta$ in $\tilde{t}$. With the above values the Cardy formula leads to

$$\Delta S = 2\pi\sqrt{\frac{4m_0k\alpha^2}{-\tilde{R}_0 lG}} \,, \tag{4.22}$$

and taking into account that

$$m_0 k\alpha^2 = m - m_0 = \Delta m \,, \tag{4.23}$$

we get

$$\Delta S = 2\pi\sqrt{\frac{4\Delta m}{-\tilde{R}_0 lG}} \,. \tag{4.24}$$

We shall now check explicitly that this expression exactly agrees with the deviation of the Bekenstein-Hawking entropy

$$S^{BH} = \frac{\mathcal{V}^{(n)} r^n}{4l^n} \,, \tag{4.25}$$

of a near-degenerate geometry from the entropy of the degenerate solution

$$S_0 = \frac{\mathcal{V}^{(n)} r_0^n}{4l^n} \,. \tag{4.26}$$

The deviation is then

$$\Delta S^{BH} = \frac{n\mathcal{V}^{(n)} r_0^{n-1}}{4l^n} \left.\frac{\partial r}{\partial\sqrt{\Delta m}}\right|_{\Delta m=0} \sqrt{\Delta m} + \mathcal{O}(\Delta m) \,. \tag{4.27}$$

4.1 Reissner-Nordström black holes

The radius of the extremal black hole is the double root of

$$U(r) = 1 - \frac{16\pi l^n m_0}{n\mathcal{V}^{(n)} r^{n-1}} + \frac{2(n-1)}{n}\frac{l^{2n}q^2}{r^{2(n-1)}} \,, \tag{4.28}$$

where

$$m_0 = \frac{n}{4}\sqrt{\frac{n}{2(n-1)}}\frac{q}{G} \,, \tag{4.29}$$

is the mass for the extremal case. Then the radius reads

$$r_0^{n-1} = \frac{8\pi l^n m_0}{n\mathcal{V}^{(n)}} \,. \tag{4.30}$$

We also get

$$\phi_0 = \frac{n}{8(n-1)} \left(\frac{n}{2(n-1)l^2 q^2} \right)^{\frac{-n}{2(n-1)}} . \tag{4.31}$$

Expanding around the extremal radius

$$r^{n-1} = \frac{8\pi l^n}{n\mathcal{V}^{(n)}} m_0 + \frac{16\pi l^n}{\sqrt{2} n \mathcal{V}^{(n)}} \sqrt{m_0 \Delta m} (1 + \mathcal{O}(\Delta m)) \,, \tag{4.32}$$

the entropy deviation (4.27), to leading order in $\sqrt{\Delta m}$, is

$$\Delta S^{BH} = 2\pi \sqrt{\frac{2r_0^2 m_0 \Delta m}{(n-1)^2}} = 2\pi \sqrt{\frac{n^2}{4(n-1)^3} \left(\frac{2(n-1)l^2 q^2}{n} \right)^{\frac{1+n}{2(n-1)}} \frac{l \Delta m}{G}} \,. \tag{4.33}$$

But this exactly coincides with the statistical entropy (4.24) since, by a straightforward computation, we have that

$$-l^2 \tilde{R}_0 = J''(\phi_0) = \frac{16(n-1)^3}{n^2} \left(\frac{n}{2(n-1)l^2 q^2} \right)^{\frac{1+n}{2(n-1)}} . \tag{4.34}$$

4.2 Schwarzschild-de Sitter black holes

Now we have

$$U(r) = 1 - \frac{16\pi l^n m_0}{n\mathcal{V}^{(n)} r^{n-1}} - \frac{2\Lambda r^2}{n(n+1)} \,. \tag{4.35}$$

To get the horizons we study the roots of the following polynomial

$$P(r) = r^{n-1} - \frac{16\pi l^n m_0}{n\mathcal{V}^{(n)}} - \frac{2\Lambda}{n(n+1)} r^{n+1} \,, \tag{4.36}$$

and we find that for $0 < m < m_0$, where

$$m_0 = \frac{n\mathcal{V}^{(n)}}{8\pi l^n} \left(\frac{n(n-1)}{2\Lambda} \right)^{\frac{n-1}{2}} , \tag{4.37}$$

there are two positive roots r_-, r_+ that become a double root r_0 in the limit $m = m_0$

$$r_0 = \sqrt{\frac{n(n-1)}{2\Lambda}} \,. \tag{4.38}$$

For $m > m_0$ there is no root. The physical picture is a black hole in an asymptotic de Sitter spacetime. r_- and r_+ are respectively the radius of the black hole and cosmological horizons. The degenerate case in which both horizons merge at r_0 is given for $m = m_0$.

Now in order to get the entropy deviation (4.27) we expand the polynomial around the degenerate radius and, taking into account that $m = m_0 + \Delta m$ ($0 \ll \Delta m < 0$) and $P(r_\pm) = 0$, we get

$$r_\pm - r_0 = \pm \sqrt{\frac{2r_0^2}{(n-1)m_0}} \sqrt{|\Delta m|} \,. \tag{4.39}$$

Then the entropy deviation (4.27) reads

$$|\Delta S_{\pm}^{BH}| = 2\pi\sqrt{\frac{n^2}{4(n-1)^2}\left(\frac{n(n-1)}{2\Lambda l^2}\right)^{\frac{n+1}{2}}\frac{l|\Delta m|}{G}}\,. \tag{4.40}$$

But now

$$-l^2\tilde{R}_0 = J''(\phi_0) = -\frac{16(n-1)^2}{n^2}\left(\frac{2\Lambda l^2}{n(n-1)}\right)^{\frac{n+1}{2}}\,, \tag{4.41}$$

where

$$\phi_0 = \frac{n}{8(n-1)}\left(\frac{n(n-1)}{2\Lambda l^2}\right)^{\frac{n}{2}}\,, \tag{4.42}$$

and (4.40) exactly coincides with the statistical entropy (4.24).

5 Conclusions and final remarks

The goal of this paper is to point out that the deviation of the Bekenstein-Hawking entropy of nearly degenerate black holes from the degenerate solution can be computed, via Cardy's formula, from the conformal asymptotic symmetry of the geometries (A)$dS_2\times S^n$ associated with the degenerate Reissner-Nordström and Schwarzschild-de Sitter black holes. Partial results has been obtained in a previous paper [19] and here we have generalized them to arbitrary dimensions and also for geometries with a dS_2 factor. We can wonder whether these results can also be further extended to other types of black holes. According to the analysis of [19], this mechanism to derive the entropy for nearly degenerate black holes works for a generic two-dimensional dilaton gravity theory. Therefore we can conclude that our approach can be applied to any higher-dimensional black hole whose thermodynamics can be effectively described by the thermodynamics of a two-dimensional dilaton theory. So, for instance, the string black holes considered in [35] are natural candidates to further extend our results.

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Appendix A Conformal redefinitions and dimensional reduction

In section 4, a conformal reparametrization (4.9), (4.10) was used in order to get the effective two-dimensional theory that describes the geometry close to the degenerate horizon. We shall now state precisely some technical aspects of it. Let us rewrite the effective action (4.8) in the form

$$I = \frac{1}{2G}\int d^2x\sqrt{-\hat{g}}\left(D(\psi)\hat{R} + H(\psi)|\nabla\psi|^2 + l^{-2}\hat{V}(\psi)\right)\,, \tag{A.1}$$

where $D(\psi)$ is given by (4.9) and

$$\begin{aligned} H(\psi) &= \frac{n^2}{8}\psi^{n-2}\,, && \text{(A.2)}\\ \hat{V}(\psi) &= \frac{n^2}{8}l^{-2}\psi^{n-2} - \frac{n(n-1)}{4}q^2\psi^{-n} - \frac{n}{4(n-1)}\Lambda\psi^n\,. && \text{(A.3)} \end{aligned}$$

In order to get (4.12) we perform the conformal redefinition (4.10) where $\phi = D(\psi)$ and

$$V(\phi) = \frac{\hat{V}(\psi(\phi))}{\Omega^2(\phi)}\,. \tag{A.4}$$

Finally $\Omega^2(\phi)$ can be obtained by means of the the following differential equation [34]

$$\frac{1}{2} - \frac{dD}{d\psi}\frac{d\ln\Omega}{d\psi} = 0\,. \tag{A.5}$$

It is

$$\Omega^2(\phi) = C\left(\frac{8(n-1)}{n}\phi\right)^{\frac{n-1}{n}}\,, \tag{A.6}$$

where C is an integration constant. The new potential (A.4) is then written

$$\begin{aligned} V(\phi) &= \frac{n^2}{8C}\left(\frac{8(n-1)}{n}\phi\right)^{\frac{-1}{n}} - \frac{n^2}{8C}\frac{l^2q^2}{\Delta_{(n)}}\left(\frac{8(n-1)}{n}\phi\right)^{\frac{1-2n}{n}} - && \text{(A.7)}\\ &\quad \frac{n}{8(n-1)}\frac{2l^2\Lambda}{C}\left(\frac{8(n-1)}{n}\phi\right)^{\frac{1}{n}}\,. \end{aligned}$$

In order to determine the constant C, recall that in (4.10) $d\hat{s}^2_{(2)}$ and $ds^2_{(2)}$ are given respectively by (4.7) and (4.15). It follows immediately that

$$J(\phi) - 2Glm = \Omega^2(\phi)U(r)\,, \tag{A.8}$$

where

$$\phi = \frac{n}{8(n-1)}\left(\frac{r}{l}\right)^n\,. \tag{A.9}$$

We get

$$\Omega^2(\phi) = \left(\frac{n^2}{8(n-1)}\right)^2 C^{-1}\left(\frac{8(n-1)}{n}\phi\right)^{\frac{n-1}{n}}\,, \tag{A.10}$$

thus comparing with (A.6) we finally obtain

$$C = \frac{n^2}{8(n-1)}\,, \tag{A.11}$$

in agreement with (4.11)

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